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Optimal Streamwise Vortices Intended for Supersonic Mixing Enhancement

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The theory of optimal, spatially growing disturbances is applied to a compressible turbulent mixing layer. It is suggested that the weak, steady disturbances generated by tiny vortex generators be considered as in the triple-decomposition method. The linearized equations for the disturbances are closed via the eddy viscosity model with a turbulent Prandtl number equal to one. The optimization procedure is formulated in terms of Mack's energy norm. As an example, a parallel flow with a hyperbolic tangent velocity profile and a temperature profile following from the Crocco–Busemann relation is analyzed. Theoretical results indicate that the optimization procedure leads to streamwise vortices and that it is consistent with experimental findings. The theory provides the possibility of estimating the spacing of the tiny vortex generators placed on the splitter plate. Results show that an increase in the convective Mach number is accompanied by an increase in the transient growth effect.

Nomenclature

a	=	speed of sound
D	=	$\mathrm{d}/\mathrm{d}y$
G	=	energy ratio parameter
M	=	Mach number
M_c	=	convective Mach number
N	=	number of Chebyshev polynomials per domain
N_m	=	number of eigenmodes used in the optimization procedure
p	=	pressure disturbance
Re_t	=	turbulent Reynolds number
T	=	temperature disturbance
T_s	=	mean flow temperature
t	=	time
U	=	mean flow velocity
u	=	streamwise velocity disturbance
v	=	transverse velocity disturbance
w	=	spanwise velocity disturbance
x	=	streamwise coordinate
y	=	transverse coordinate
$\pm y_{\max}$	=	boundaries for the problem formulation
α	=	streamwise wavenumber
β	=	spanwise wavenumber
γ	=	specific heat ratio
ρ_s	=	mean flow density
μ_t	=	turbulent flow viscosity

Subscript

1/2	=	upper/lower flow ($U_1 > U_2$)

Superscripts

T	=	transposed
$'$	=	random field (fine-grained turbulence)
$\hat{}$	=	introduced three-dimensional steady disturbance

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Introduction

TECHNOLOGICAL applications, such as supersonic combustion, supersonic ejectors, and noise suppression, require efficient mixing in compressible shear layers.[1,2] The governing parameter characterizing the compressibility effects is the convective Mach number, which, in a case with the gases having the same ratio of specific heats in both streams, might be evaluated as $M_c = (U_1 - U_2)/(a_1 + a_2)$, where U_1 and U_2 are the streamwise velocities and a_1 and a_2 are the speeds of sound in the two different streams. The subscript 1 stands for the high-speed flow. As the convective Mach number increases, the spreading rate of the shear layer decreases and, at $M_c > 0.8$, the mixing is strongly suppressed.

One promising approach to mixing enhancement is to introduce streamwise vortices into the flow. Foss and Zaman[3] and Zaman[4] investigated the effects of tabs and other nozzle geometry modifications. Krothapalli et al.,[5] King et al.,[6] and Krothapalli et al.[7] showed that streamwise vortices could be generated by tiny three-dimensional roughness elements placed into the boundary layer near the splitter tip. Experiments by Island et al.[8] showed that the sub-boundary elements can lead to an increase of the spreading rate in planar compressible shear layers, as well. The latter results are considered as promising, but were obtained only for a specific convective Mach number, $M_c = 0.63$, and for a few geometries of the roughness elements. Bourdon and Dutton[9] applied the technique to a base flow. To optimize the vortex generators and to gain the most benefit from this technology, a theoretical model for analysis of the streamwise vortices is required.

The problem of optimal disturbances has been of great interest during the past decade in the context of bypass transition to turbulence. There are many applications where transition to turbulence occurs without exponentially growing ("modal growth") disturbances, such as Tollmien–Schlichting waves (see Ref. 10). As follows from theoretical and experimental data, there is great potential for disturbance energy growth, even if the flow is stable with respect to wavelike perturbations. The theory and experiments predict that the optimal disturbances, leading to a significant effect on the base flow, are stationary streamwise vortices.[10] These vortices should be properly spaced in the spanwise direction to provide the necessary effect on the shear flow. It means that an analysis of optimal disturbances might be helpful in predicting the spanwise periodicity of the streamwise vortex generators to provide the strongest effect on the basic flow.

The problem has been considered in a number of papers concerning compressible and incompressible near-wall flows (boundary layer, pipe, channel flows, etc.).[11–15] One can find a vast bibliography in a monograph by Schmid and Henningson.[16] The main theoretical results related to the optimal disturbances have been obtained within the framework of temporal theory, whereas a spatial development is observed in experiments. The difficulty associated with the spatially growing disturbances originates from the ill posedness of the initial-value problem for linearized Navier–Stokes equations. To overcome this difficulty, Luchini[14] and Andersson et al.[15] considered a spatial growth in boundary layers within the scope of the linearized boundary-layer equations. A consistent spatial formulation for optimal disturbances has been proposed recently.[17–19] The objective of the present work is to apply the theory of optimal disturbances to the problem of mixing enhancement in compressible turbulent mixing layers.

Governing Equations

The steady perturbations in a turbulent compressible mixing layer will be analyzed in a manner similar to the triple-decomposition method[20] used for analysis of large coherent structures in turbulent shear flows. The instantaneous flowfield in a turbulent flow is presented as a sum of two components:

$$q(x, y, z, t) = \bar{q}(x, y, z) + q'(x, y, z, t) \tag{1}$$

where $\bar{q}$ is the mean (time-averaged) value of a parameter (velocity, pressure, and temperature) and q' stands for the random (turbulent) motion. After substitution into the Navier–Stokes equations and time averaging, the equations for the mean flow are obtained and are simplified by applying the boundary-layer approximation. It is well known that the equations are not closed. They contain terms such as $\overline{u'v'}$ and $\overline{v'T'}$, and a closure hypothesis should be adopted. Further decomposition is associated with the mean values

$$\bar{q}(x, y, z) = Q(x, y) + \hat{q}(x, y, z) \tag{2}$$

where Q is the undisturbed mean flow and $\hat{q}$ stands for the weak three-dimensional disturbances introduced into the planar mixing layer. The latter decomposition leads to the equations for the disturbances $\hat{q}$, where the nonlinear terms can be neglected under the assumption of weak perturbations. The procedure generates new unknown terms, originated as disturbances of the terms $\overline{u'v'}$ and $\overline{v'T'}$. These terms describe the interaction of introduced three-dimensional stationary disturbances with fine-grained turbulence, and a closure hypothesis should be applied.

This approach, as already mentioned, originates from the triple-decomposition method. However, instead of the coherent periodic-in-time component, steady three-dimensional perturbations are introduced. Reynolds and Hussain[20] suggested that the equations for the coherent component might be closed with the Newtonian eddy viscosity model, where the eddy viscosity corresponds to the undisturbed flow. The model was used in various applications for the analysis of periodic signals in turbulent flows. Marasli et al.[21] and Reau and Tumin[22,23] analyzed harmonic perturbations in low-speed turbulent wakes and demonstrated very good agreement with experimental data. Reau and Tumin[24] also applied the model to the analysis of periodic signals in a turbulent mixing layer and obtained very good agreement with experimental data. The same ideas of triple decomposition and closure via the eddy viscosity of the undisturbed flow have been used for the analysis of a compressible turbulent mixing layer.[25]

In the present work, closure via the eddy viscosity of the undisturbed flow is adopted. The assumption leads to the conventional stability equations in a compressible shear flow with turbulent Reynolds and Prandtl numbers. The equations with proper boundary conditions allow an optimal analysis of the streamwise vortices similar to the procedure used for laminar near-wall flows.

The solution of the linearized equations for the stationary coherent disturbances is considered in the form $\sim e^{i(\alpha x + \beta z)}$, where the transverse wave number β is prescribed. The equations are reduced to a system of ordinary differential equations in the following form[19]:

$$(AD^2 + BD + C)\Phi = 0 \tag{3}$$

where $\Phi = [\hat{u}(y), \hat{v}(y), \hat{p}(y), \hat{T}(y), \hat{w}(y)]^T$ is a five-element vector comprising the complex amplitude functions, x and y components of velocity, pressure, and temperature and the z component of velocity. (The superscripted T stands for the vector transpose.) A, B, and C are 5×5 matrices, and $D = \mathrm{d}/\mathrm{d}y$. The boundary conditions for Eq. (3) are

$$y \to \pm\infty : |\Phi_j| < \infty \tag{4}$$

The boundary conditions (4) allow decaying eigenmodes ($y \to \infty : \Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5 \to 0$) that represent the discrete spectrum and nongrowing, nondecaying modes that represent the continuous spectrum.

Numerical Method

Eigenmodes of the problem [Eqs. (3) and (4)] were solved with the two-domain Chebyshev spectral collocation method (see Ref. 26). The boundary condition (4) is replaced by

$$y \to \pm y_{\max} : \Phi_j = 0 \tag{5}$$

The latter boundary condition leads to discretization of the continuous spectra and leaves the discrete modes untouched if the boundaries $\pm y_{\max}$ are chosen sufficiently far away.

The following vector function is introduced for the purpose of analyzing a spatially growing disturbance:

$$\tilde{\Phi} = [\hat{u}, \hat{v}, \hat{p}, \hat{T}, \hat{w}, \alpha\hat{u}, \alpha\hat{v}, \alpha\hat{T}, \alpha\hat{w}]^T \tag{6}$$

and Eq. (3) is recast in such way that the new system contains α only in the first power.

The Chebyshev spectral collocation method is applied to the domains $-y_{\max} \le y \le 0$ and $0 \le y \le y_{\max}$. Algebraic stretching[26] is employed for both physical domains to transform them to the computational domain $-1 \le \xi \le 1$. For example, the upper domain is transformed in accordance with

$$y = d(1 + \xi)/(b - \xi) \tag{7}$$

where $b = 1 + 2d/y_{\max}$ and $d = y_i y_{\max}/(y_{\max} - 2y_i)$. The parameter y_i is chosen to locate half of the grid points in the interval $(0, y_i)$. The Nth-order Chebyshev polynomials T_N are used with the collocation points

$$\xi_j = \cos(\pi j/N), \qquad j = 1, \ldots, N \tag{8}$$

The continuity condition for the functions and their first derivatives at $y = 0$ are applied as the matching condition for the solutions in the two domains.

The resulting system of algebraic equations leads to the generalized eigenvalue problem

$$A\tilde{\Phi} = \alpha B\tilde{\Phi} \tag{9}$$

where A and B are $18N \times 18N$ matrices. The eigenvalue problem is solved with the help of a standard routine, DG6CCG, from the IMSL FORTRAN Library.

Spatial Theory of Optimal Disturbances

The spatial theory of optimal disturbances within the scope of linearized Navier–Stokes equations was developed in Refs. 17–19. For the sake of consistency and continuity, the milestones of the method are recapitulated briefly.

The continuous spectrum of the linearized Navier–Stokes equations in an unbounded area has seven branches.[27,28] As follows from the analysis of the signaling problem, four branches are responsible for the flow response downstream of the disturbance source, whereas the other three branches enter into the flow response upstream of the source. This classification of the branches of the continuous spectrum is a crucial element of the theory because it allows formulation of an initial-value problem for the spatially growing disturbances.

A vector function $\boldsymbol{q} = (\hat{u}, \hat{v}, \hat{\rho}, \hat{T}, \hat{w})^T$ and a scalar product

$$(\boldsymbol{q}_1, \boldsymbol{q}_2)_2 = \int_{-\infty}^{\infty} \boldsymbol{q}_1^H \boldsymbol{L} \boldsymbol{q}_2 \, \mathrm{d}y \tag{10}$$

are introduced, where the matrix $\boldsymbol{L}$ is

$$\mathrm{diag}\left\{\rho_s,\ \rho_s,\ T_s\big/\left(\gamma\rho_s M_1^2\right),\ \rho_s\big/\left[\gamma(\gamma-1)T_s M_s^2\right],\ \rho_s\right\}$$

and H is the complex conjugate transpose.

The definition of the scalar product leads to the energy norm

$$\|\boldsymbol{q}\|^2 = (\boldsymbol{q}, \boldsymbol{q})_2 = \int_{-\infty}^{\infty} \boldsymbol{q}^H \boldsymbol{L}\boldsymbol{q}\,\mathrm{d}y \tag{11}$$

This energy norm was first introduced by Mack[29] and later rederived by Hanifi et al.[30] Following the optimization procedure by Schmid and Henningson,[31] the vector function $\boldsymbol{q}$ is expanded into N_m decaying eigenmodes (they might belong to the discrete spectrum or to the numerical discretization of the continuous spectrum):

$$\boldsymbol{q}(x, y, z) = e^{i\beta z}\sum_{k=1}^{N_m} \kappa_k \hat{\boldsymbol{q}}_k(y) e^{i\alpha_k x} \tag{12}$$

The vector function $\hat{\boldsymbol{q}}_k$ comprises three velocity components, density, and temperature corresponding to the kth eigenfunction. The coefficients κ_k are to be optimized to achieve the maximum energy growth at the specific downstream coordinate

$$G(\beta, R_t, x) = \max\left(\|\boldsymbol{q}(x)\|^2/\|\boldsymbol{q}_0\|^2\right) \tag{13}$$

where $\|\boldsymbol{q}_0\|^2$ is the energy at the initial location $x = 0$. As follows from the method in Ref. 31, the latter may be calculated as the 2 norm of a matrix:

$$G(\beta, R_t, x) = \left\|\boldsymbol{F}\boldsymbol{\Lambda}_x\boldsymbol{F}^{-1}\right\|_2^2 \tag{14}$$

where $\boldsymbol{\Lambda}_x = \mathrm{diag}(e^{i\alpha_1 x}, e^{i\alpha_2 x}, \ldots, e^{i\alpha_{N_m} x})$ and the matrix $\boldsymbol{F}$ is defined with help of the matrix $\boldsymbol{A}$ as follows:

$$\boldsymbol{F}^H\boldsymbol{F} = \boldsymbol{A}, \qquad A_{k,l} = (\boldsymbol{q}_k, \boldsymbol{q}_l)_2 \tag{15}$$

The 2 norm of the matrix $\boldsymbol{F}\boldsymbol{\Lambda}_x\boldsymbol{F}^{-1}$ can be determined by using a singular value decomposition, and it is equal to the largest singular value σ_1 (Ref. 31).

Results

The locally parallel flow approximation is adopted to estimate the potential of nonmodal growth in the compressible mixing layer. The undisturbed mean velocity profile is approximated as

$$U(y)/U_1 = \eta(y) + (U_2/U_1)[1 - \eta(y)]$$

$$\eta(y) = \tfrac{1}{2}[1 + \tanh(y)] \tag{16}$$

and the temperature profile conforms to the Crocco–Busemann relation. In this specific case, where $U_2 = 0$ and the freestream Mach number $M_1 = 2$, the convective Mach number can be evaluated as

$$M_c = M_1/1 + \sqrt{T_2/T_1} \tag{17}$$

In these calculations, the turbulent Prandtl number was equal to 1, and the eddy viscosity coefficient μ_t was chosen as a linear function of the undisturbed temperature T_s. The latter follows from the hypothesis of Alber and Lees.[32] Figure 1 shows the eigenvalue maps corresponding to $T_2/T_1 = 0.5$, $Re_t = 30$, and $\beta = 0.3$. Figure 1a is a comparison of the boundary $y_{\max}$ effect when the number of Chebyshev polynomials N in each domain is equal to 51. Figure 1b shows a comparison of the eigenvalues obtained at $N = 51$ and 101. In all numerical results, the inner point $y_i = 2.0$. For the present evaluations, $N = 51$.

Figure 2 shows the energy ratio G vs the spanwise wave numbers $\beta = 2\pi/\lambda_z$ at a specific finite distance x, downstream from the generator. The results indicate that, for a specific distance x, there is an optimum spacing λ_z of the disturbances (vortex generators).

Results in Fig. 2 were obtained at the turbulent Reynolds number $Re_t = 30$. Note that the turbulent Reynolds number is not a free parameter. It depends on the spreading rate of the mixing layer.[24] The latter is a function of the velocity ratio and the convective Mach number.[1] For the present estimations of the transient growth effect, we choose the turbulent Reynolds number without a link to the velocity ratio and the convective Mach number because previous investigations[16] have shown that the energy growth G is scaled with the second power of the Reynolds number. Figure 3 demonstrates the scaling law for the mixing layer ($Re_t = 30$ and 300). The latter means that results presented in accordance with the scaling law (as it is in Figs. 2 and 3) are independent of the specific value of Reynolds number Re_t. Figure 4 shows about 200 modes used in the optimization procedure at $Re_t = 300$.

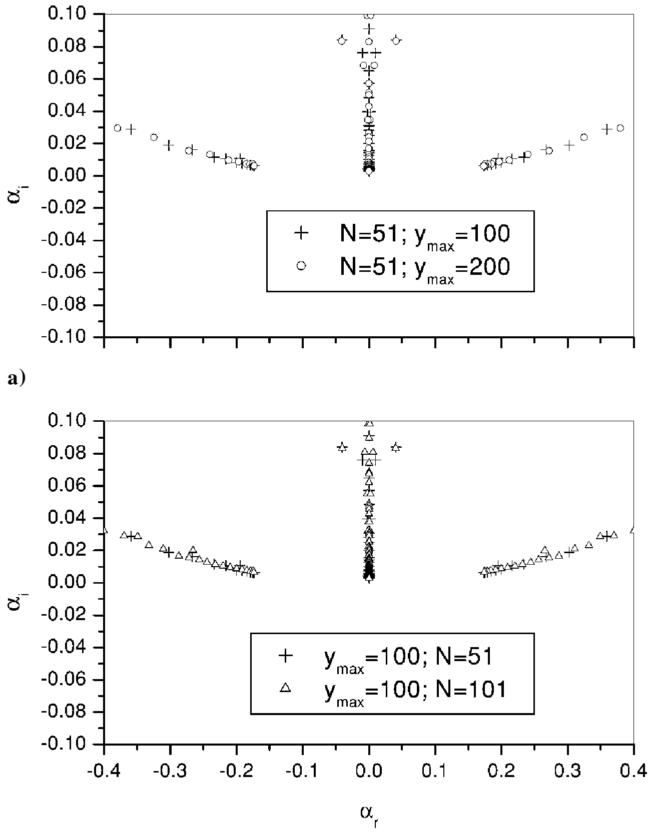

Fig. 1 Eigenvalue map for $M_1 = 2$, $T_2/T_1 = 0.5$, $Re_t = 30$, $y_i = 2$, and $\beta = 0.3$: a) effect of the outer boundary and b) effect of the number of polynomials.

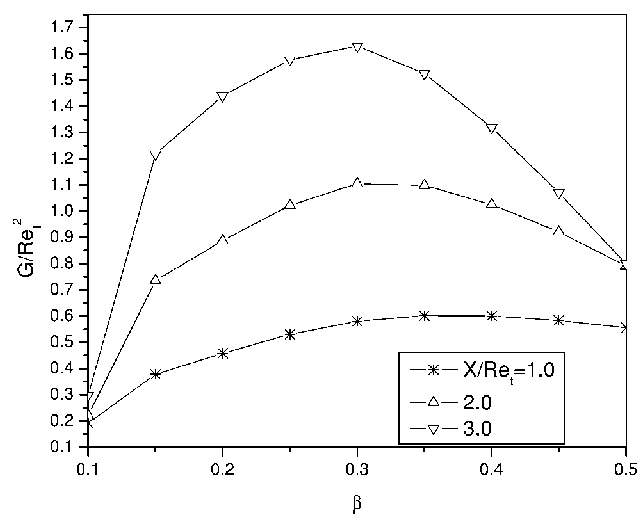

Fig. 2 Effect of the spanwise wave number on the energy ratio; $M_1 = 2$, $T_2/T_1 = 0.5$, and $Re_t = 30$.

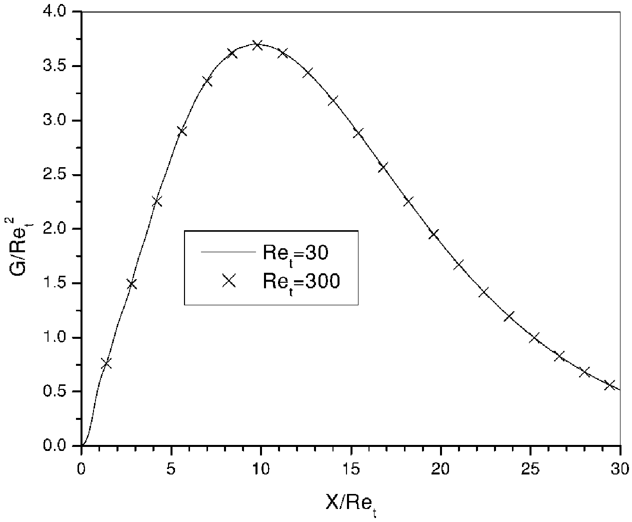

Fig. 3 Scaling law for a compressible mixing layer; $M_1 = 2$, $T_2/T_1 = 0.5$, and $\beta = 0.3$.

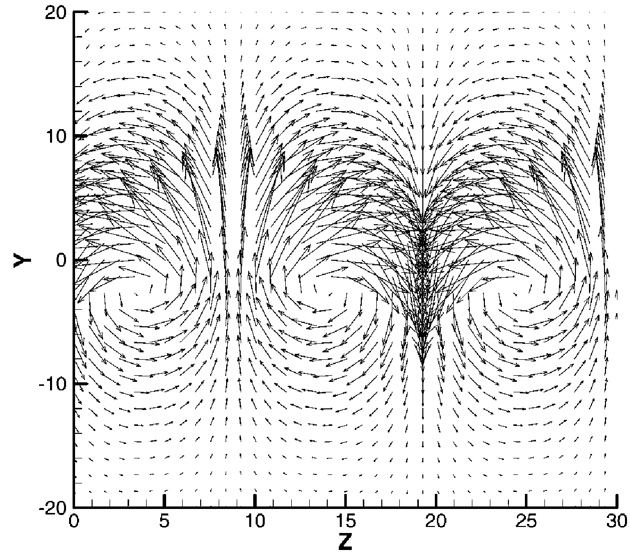

Fig. 6 Flowfield in the y, z plane at $X/Re_t = 10$; $M_1 = 2$, $T_2/T_1 = 0.5$, $Re_t = 30$, $\beta = 0.3$, and $X/Re_t = 10$.

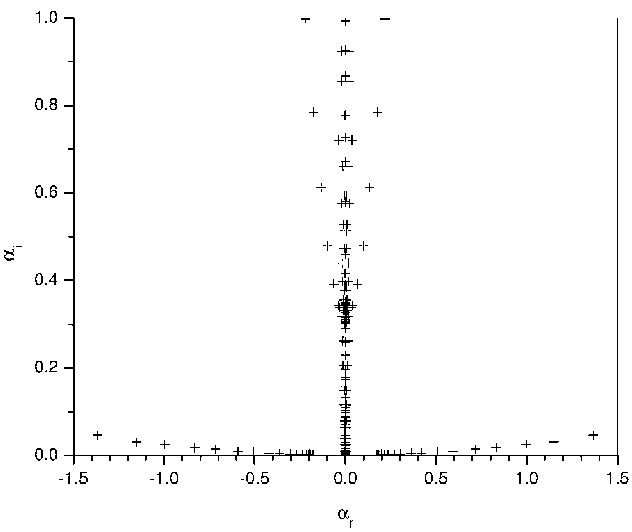

Fig. 4 Eigenvalue map for $M_1 = 2$, $T_2/T_1 = 0.5$, $Re_t = 30$, and $\beta = 0.3$.

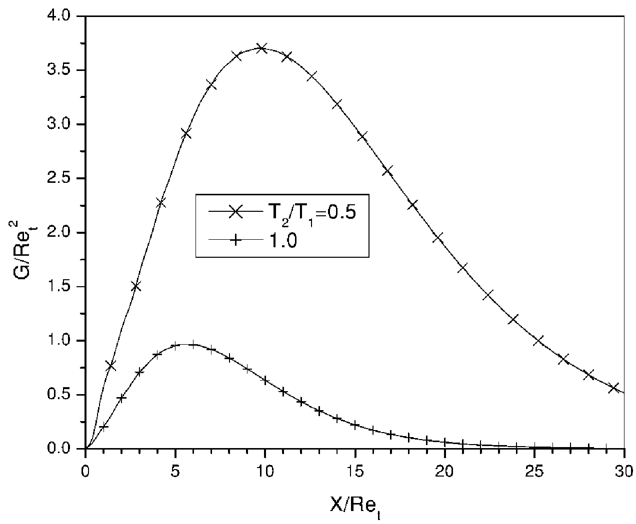

Fig. 7 Effect of the temperature ratio (convective Mach number) on the transient growth; $M_1 = 2$ and $\beta = 0.3$.

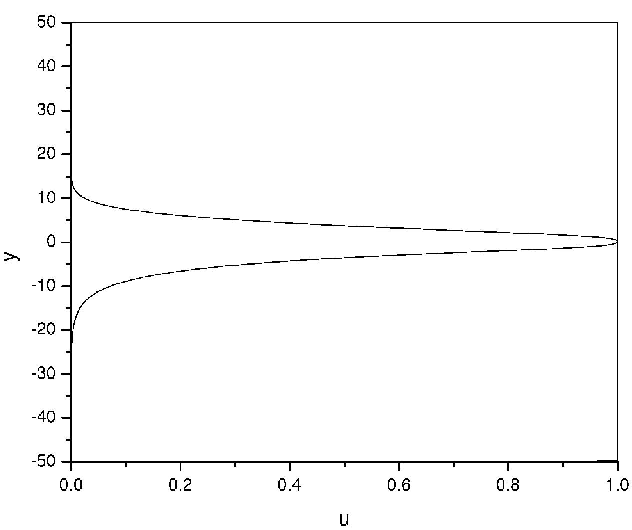

Fig. 5 Streamwise velocity disturbance at $X/Re_t = 10$, $M = 2$, $T_2/T_1 = 0.5$, $\beta = 0.3$, and $Re_t = 30$.

The streamwise velocity disturbance profile at $X/Re_t = 10$ is presented in Fig. 5. The flowfield in the y, z plane at this location is shown in Fig. 6. The optimal analysis led to the steady streamwise vortices accompanied by the streaky structures in the velocity profile.

The effect of the temperature ratio, T_2/T_1, and, therefore, the convective Mach number (17) is shown in Fig. 7. The results indicate that the increase of the convective Mach number is accompanied by an increase in the transient growth effect.

Conclusions

The objective of the present work was to apply the theory of optimal disturbances to the problem of a supersonic mixing enhancement. The theoretical results indicate that the optimization procedure leads to streamwise vortices and that it is consistent with experimental findings. The theory, in its simplest (parallel flow) approximation, provides the possibility of estimating the optimal spacing of the tiny vortex generators placed on the splitter plate. The optimal shape of the generators might be derived from the receptivity problem solution linked to the optimal disturbances (streamwise vortices).

In conclusion, two drawbacks of the model that should be addressed in future research are noted:

1) An adequate description of the flowfield must include the nonparallel flow effects, and the latter might be incorporated into the model by considering the linearized boundary-layer equations as was implemented for a Blasius boundary layer in Refs. 14 and 15.

2) The closure model for disturbances in a turbulent mixing layer has been justified by comparison with experimental data in Refs. 22–24 for traveling waves. Applying the same model for the stationary disturbances is questionable, and further experimental and computational investigations will be helpful.

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